Sequences and Series 4

1. By using the difference method find the sim of the first n terms:

$$\frac{1}{1\cdot2\cdot3\cdot4}+\frac{1}{2\cdot3\cdot4\cdot5}+\frac{1}{3\cdot4\cdot5\cdot6}+\cdots=\frac{1}{18}-\frac{1}{3(n+1)(n+2)(n+3)}$$

2. Express $\frac{2}{r^2-1}$ in partial fractions. Hence, find a simple expression for $S=\sum_{r=2}^{n}\frac{2}{r^2-1}$ and determine whether S converges when n tends to infinity.

3. If $\sum_{r=1}^{n}T_r=3n^2+4n$, find the value of $\sum_{r=1}^{n-1}T_r$.
Deduce the general term T_n and hence find $\sum_{r=n+1}^{2n}T_r$.

4. The sum of the first 2n terms of a series $18n-12n^2$. Find the sum of the first n terms and the nth term of this series. Show that this series is an arithmetic.

5. In a set of integers between the numbers 1 and 10,000,

(a) how many of these numbers are divisible by 3,4, 5 and 11?

(b) how many of these numbers are divisible by 3,4, 5 or 11?

6. The sum of the first n terms of a geometric sequence is given by $S_n=15(1-3^{-n})$. Find

(a) the nth term,

(b) the common ratio,

(c) the smallest value of n such that $S_\infty-S_n<0.01$.

7. Verify the identity $\frac{2r-1}{r(r-1)}-\frac{2r+1}{r(r+1)}=\frac{2}{(r-1)(r+1)}$. Hence, using the method of difference, prove that

$$\sum_{r=2}^{n}\frac{2}{(r-1)(r+1)}=\frac{3}{2}-\frac{2n+1}{n(n+1)}\ .$$

Deduce the sum of the infinite series $\frac{1}{1\times3}+\frac{1}{2\times4}+\frac{1}{3\times5}+\cdots+\frac{1}{(n-1)(n+1)}+\cdots$

8. If $\frac{1}{x},\frac{1}{y},\frac{1}{z}$ are three consecutive terms in an arithmetic progression.

Show that $\frac{y+z}{x},\frac{z+x}{y},\frac{x+y}{z}$ also from three consecutive terms in an arithmetic progression.

9. If S is the sum of the series $1 + 3x + 5x^2 + 7x^3 + \cdots + (2n+1)x^n$, for $x \neq 1$, by considering $(1-x)S$, or otherwise, show that $S = \frac{1+x-(2n+3)x^{n+1}+(2n+1)x^{n+2}}{(1-x)^2}, x \neq 1$.

10. (a) The sequence of positive integers is grouped into four as follows:

$(1,2,3,4), (5,6,7,8), (9,10,11,12), \ldots$

Show that the sum of all integers in the k^{th} bracket is $S_k = 2(8k-3)$.

(b) If the integers are similarly grouped with m integers in each bracket, find the sum S_n of all integers in the n^{th} bracket in terms of m and n.

Hence, show that S_n, S_{2n}, S_{3n} are in arithmetic progression.